

Pomeron Vertices in Perturbative QCD in Diffractive Scattering

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Abstract: We analyse the momentum space triple Pomeron vertex in perturbative QCD. In addition to the standard form of this vertex which is used in the context of total cross-sections at high energies and in the QCD reggeon field theory, there exists an alternative form which has to be used in the study of high-mass diffraction. We review and analyse the relation between these two versions. We discuss some implications for the BK-equation. In the second part of our paper we extend this analysis to the Pomeron-Odderon-Odderon vertex.

1 Introduction

It is well known that the BFKL Pomeron - which in perturbative QCD describes the high energy elastic scattering of small-size projectiles - is part of an effective 2+1 dimensional field theory. Apart from the BFKL kernel (and its generalization to the kernel of the BKP equation), the most essential element of this field theory, known so far, is the vertex which describes the transition of two to four reggeized gluons; as the most important application, it leads to the splitting of one BFKL Pomeron into two Pomerons (PPP vertex). This vertex was first derived in the study of high-mass diffractive scattering of two virtual photons [1, 2]. The triple Pomeron vertex appears after summing and integrating over the diffractive final states and removing a certain piece which can be associated with a single Pomeron exchange (see some details below). In the context of the 2+1 dimensional field theory, this vertex can then be used to compute, for example, the Pomeron self-energy. Another application is the evolution equation for the sum of the fan diagrams which, in the large- N_c limit, coincides with the Balitsky-Kovchegov (BK) equation [3, 4, 5]. Also, this vertex easily allows to understand the AGK counting rules in pQCD. On the other hand, in the Regge-Gribov formalism high-mass diffraction is standardly ascribed exclusively to the triple Pomeron interaction. This suggests that there may exist an alternative form of the triple Pomeron vertex, the diffractive PPP vertex, suitable for the description of high-mass diffraction.

To illustrate this difference it is useful to recapitulate briefly the derivation of the former PPP-vertex. Ref.[2] introduces amplitudes which describe the coupling of virtual photons to four t -channel gluons, named reggeon-particle amplitudes, D_4 . They are derived from multiple energy discontinuities and include the integration over the rhs cuts of the energy variables. One starts from gluon production amplitudes with exchanges of reggeized gluons, and then sums over the number of produced gluons and integrates over the produced mass. In the language of angular momentum theory, however, this function D_4 is not yet a t -channel

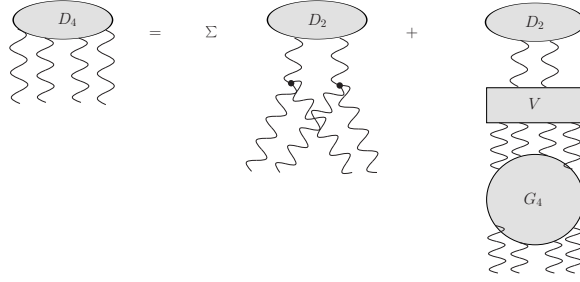


Figure 1: Decomposition for $D_4 = D_4^R + D_4^I$.

partial wave, since the lhs energy-cuts have not yet been included, and signature has not been defined. As outlined in [2], the final step for obtaining signatured partial waves is the decomposition of D_4 into pieces with definite symmetries with respect to the interchange of reggeized t -channel gluons. In particular, it was observed that there are pieces of D_4 which are antisymmetric under the exchange of two gluons and satisfy bootstrap equations. As a result of these bootstrap equations, these pieces of D_4 can be reduced to BFKL amplitudes, D_2 , with two reggeized gluons only. The sum of all these reggeizing pieces has been named D_4^R (the superscript 'R' refers to 'reduction'). After subtracting D_4^R from D_4 , one is left with a new amplitude, D_4^I , which is totally symmetric under the interchange of the outgoing gluons:

$$D_4(1, 2, 3, 4) = D_4^R(1, 2, 3, 4) + D_4^I(1, 2, 3, 4). \quad (1)$$

This amplitude D_4^I now has all the properties of the partial wave, and it contains the $2 \rightarrow 4$ gluon vertex V which can be used to define the triple Pomeron vertex. This vertex is one of the fundamental interaction terms in the BFKL field theory. As outlined in [6, 7], reggeon field theory is, by construction, a solution to the set of t -channel reggeon unitarity equations, and these equations require the t -channel partial waves to have the correct symmetry and partial wave properties.

The decomposition (1), in a diagrammatic fashion, is illustrated in Fig. 1. A striking feature is the absence of the double Pomeron exchange (Fig. 2) on the rhs of eq.(1), which has been included in the definition of D_4 on the lhs. At first sight, the disappearance of this contribution appears to be somewhat strange. A closer look at the discussion presented in [2], in particular at the form of the vertex V , shows that most of the double Pomeron exchange has been absorbed into the 3P interaction part D_4^I : when analysing the momentum structure of the vertex V , one finds pieces that do not correspond to an s -channel intermediate state but rather to a virtual correction. This is quite analogous to the LO BFKL kernel which consists of a 'real' piece, corresponding to the production of an s -channel gluon, and a virtual piece, coming for the gluon trajectory. As consequence of these virtual corrections inside the vertex V , D_4^I contains most of the double Pomeron exchange; some remainders have also been absorbed into D_4^R . At very high rapidities, D_4^I dominates, since it involves two Pomerons coupled to the target: the decomposition (1), therefore, also allows to separate the leading term of the amplitude D_4 in the high-energy limit.

Nevertheless, the decomposition (1) has a certain drawback. If one studies high-mass diffraction in pQCD, multigluon intermediate states are contained in both pieces in (1), i.e. this decomposition does not clearly separate the diffractive production of $q\bar{q}$, $q\bar{q}g$, $q\bar{q}gg\ldots$ final states. As mentioned, we are used to write the diffractive cross section formula in terms of a triple Pomeron coupling. Thus one might suspect that there is another, diffractive triple Pomeron vertex, which completely describes high-mass diffraction but perhaps is less

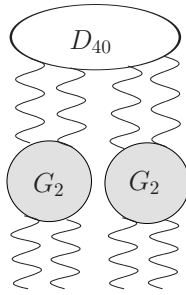


Figure 2: Double Pomeron exchange (G_2 is the BFKL Pomeron Green's function).

convenient for the study of the high-energy limit. In fact, as was already noticed in [2], the definition of the 3P vertex is not unique. A different form of the 3P vertex was later proposed in [8] and analysed in [9], which precisely corresponds to the definition of such a ‘diffractive vertex’. The idea was, starting from the same D_4 and using the large N_c limit, to find an alternative decomposition, in which, instead of the reggeizing term D_4^R , the double Pomeron exchange is separated. It turns out that the remainder then, again, contains a triple Pomeron interaction, which was named Z and differs from the previous version, V . In contrast to (1), such a decomposition no longer picks out the leading part of the amplitude in the high-energy limit, since the double P exchange and 3P part behave similarly. However, for the description of exclusive high-mass diffractive processes such a decomposition is much more natural: the first term contains only the $q\bar{q}$, but no multigluon intermediate states, whereas the second term has all the diffractive multigluon intermediate states, quite in accordance with the physical picture.

This diffractive vertex Z was first derived in [2], in the direct calculation of the high-mass diffractive cross-section at $t = 0$. In the lowest order of α_s , the cross-section for $q\bar{q}g$ production, integrated over the quark and gluon momenta, can be written in terms of the diffractive vertex Z which couple to elementary t -channel gluons. So the 3P vertex Z contains only real gluon production (and no virtual corrections)¹. This is in contrast to the triple Pomeron vertex V which has been discussed above.

All these considerations remain valid also for more complicated cases, in particular for diffractive processes governed by the exchange of Odderons. The simplest case, the diffractive production of $q\bar{q}$ states (with suitable quantum numbers), can be described by the exchange of two Odderons which in pQCD are bound states of three reggeized gluons. Then comes the production of $q\bar{q}g$, $q\bar{q}gg$,... states, and, as shown in [10], after summation over the number of gluons and integration over their momenta one arrives at an amplitude D_6 with 6 reggeized t -channel gluons and at a decomposition, analogous to (1). As a part of this decomposition, a vertex for the transition $2 \rightarrow 6$ gluons, the Pomeron→Odderon-Odderon (POO) vertex was found [10]. Here, again, the question arises whether a ‘diffractive’ version of this vertex exists and what form it has.

In the present paper, after clarifying first the relation between the vertices V and Z for the triple Pomeron case, we generalize the argument to the transition $P \rightarrow OO$ and derive the diffractive POO vertex Z . Following the derivation of Z in [2], one can derive this vertex by studying, in the lowest order, the integrated gluon production cross section of the process

¹This statement does not preclude that certain pieces of Z can be expressed in terms of the gluon trajectory function: this feature is well known from the BFKL bootstrap equation, where parts of the real gluon production inside the BFKL kernel can be expressed in terms of the gluon trajectory function

$\gamma^* q \rightarrow (q\bar{q}g)q$ where the outgoing diffractive state has C -parity opposite to the incoming photon. In this paper, however, we propose a simpler method which, in the large N_c limit, allows to avoid these calculations and readily leads to the desired diffractive vertex.

This paper is organized as follows. In section 2 we review the discussion of the PPP vertex, and we re-derive the explicit form of the ‘diffractive’ PPP vertex. In section 3 we turn to the Odderon case and obtain the diffractive POO vertex. A few details are put into 2 appendices. For simplicity, throughout the paper we will restrict ourselves to the large- N_c limit.

2 Diffractive vertex for the splitting $P \rightarrow PP$

2.1 Definitions

The diffractive vertex is derived from the high energy limit of the diffractive cross section in perturbative QCD [1, 2, 8]. To be definite, one considers the process $\gamma^* + q \rightarrow (q\bar{q} + n g) + q$ in the triple Regge limit $Q^2 \ll M^2 \ll s$, where Q^2 , M^2 and s denote the squared mass of the virtual photon, the squared mass of the diffractively produced system, and the square of the total energy, resp. The cross section formula for this process defines amplitudes with four reggeized gluons in the t -channel, D_4 . To explain our notation, we remind of the BFKL amplitude, D_2 , which appears in the elastic scattering process $\gamma^* + q \rightarrow \gamma^* + q$: it describes the exchange of two reggeized gluons, and it satisfies the BFKL equation [11] which we write in the operator form

$$S_2 D_2 = D_{20} + V_{12} \otimes D_2, \quad (2)$$

where $S_2 = j - 1 - \omega(1) - \omega(2)$ is the two-gluon ‘‘free’’ Schrödinger operator for the energy. Here $\omega(k)$ is the gluon Regge trajectory function, and V_{12} is the BFKL interaction kernel (containing only the real gluon production). Here the subscripts ‘1’, ‘2’ refer to gluon ‘1’, ‘2’ with momenta k_1 , k_2 , and color a_1 , a_2 . We suppress momentum variables and color indices. Powers of the coupling g^2 and color tensors are contained in V_{12} and will not be written explicitly. Furthermore, we simplify by writing

$$D_{20}(1) \equiv D_{20}(1, 2), \quad (3)$$

since our discussion will be done for fixed momentum transfer r^2 , and the variables for the second gluon follow from momentum conservation $k_2 = r - k_1$ etc.

To define the ‘diffractive triple Pomeron vertex’ we start from the integral equation for D_4 [2] which sums the diagrams for the diffractive production of a $q\bar{q}$ pair plus n gluons. The equation is illustrated in Fig. 3a: we take the total transverse momentum of all four gluons equal to zero, but we keep in mind that each of the gluon pairs may have a nonzero total momentum q and $-q$, with $t = -q^2$ being the invariant momentum transfer squared in the diffraction process. Each vertex corresponds to real gluon production, and all t -channel gluon lines denote reggeized gluons. As a result, one readily understands the content of this equation in terms of s -channel intermediate states. Next, using the bootstrap properties of D_3 the second and third group of terms can be combined to an effective vertex, \tilde{V} :

$$S_4 D_4 = D_{40} + \tilde{V} \otimes D_2 + U \otimes D_4, \quad (4)$$

where S_4 is defined in analogy with S_2 (after Eq.(2)), and U denotes the interactions between the lower gluons:

$$U = V_{12} + V_{34} + V_{13} + V_{14} + V_{23} + V_{24}. \quad (5)$$

Figure 3 consists of two parts, (a) and (b), each showing a graphical equation. Part (a) shows the definition of D_4 as a sum of diagrams: $(j-1-\sum_i \omega(i)) D_4$ equals D_{40} plus D_2 plus a sum over D_3 plus a sum over D_4 with indices i and j . Part (b) shows the same equation after color projection and large N_c limit, where D_4 is replaced by D_4^∞ , D_2 is replaced by D_2 with a box labeled Z below it, and the sum over D_4 is replaced by $\sum_{L,R} D_4^\infty$ with labels L and R below it.

Figure 3: (a) Graphical illustration of the definition of D_4 ; (b) the same equation after color projection and large N_c limit.

Since, in our diffractive cross section, at the lower end the two pairs of gluons (1,2) and (3,4) have to be in color singlets, we introduce a suitable color projector P which acts on the color indices of the lower gluons. In the limit of large N_c we use

$$PU \otimes D_4 \approx PU_0 \otimes PD_4, \quad (6)$$

where

$$U_0 = V_{12} + V_{34} \quad (7)$$

denotes the rungs inside the pair (1,2) and (3,4). Introducing the notation $D_4^\infty = \lim_{N_c \rightarrow \infty} PD_4$ the integral equation becomes (Fig. 3b):

$$S_4 D_4^\infty = PD_{40} + Z \otimes D_2 + PU_0 \otimes D_4^\infty. \quad (8)$$

The equation has been derived in [8]. $Z = \lim_{N_c \rightarrow \infty} P\tilde{V}$ stands for the 'diffractive triple Pomeron vertex': it describes the coupling of the two lower gluon ladders to the diffractive system. In an operator notation, it acts on the forward BFKL function D_2 and transforms the t -channel two gluon state into the diffractive colour state of four gluons. Note the explicit appearance of the double Pomeron exchange in the inhomogeneous term: when solving the integral equation by iteration, we generate the two Pomeron ladders (Fig.2) for the diffractive $q\bar{q}$ intermediate state.

Let us now discuss the connection of this vertex Z with the triple Pomeron vertex V obtained in [2]. This analysis will help us to find the explicit form of the diffractive vertex Z , and, later on, the same method can be applied to the Odderon case. Starting from the result of [2]

$$D_4 = D_4^R + D_4^I, \quad (9)$$

we remind that the second part, D_4^I , satisfies the equation (Fig. 4)

$$S_4 D_4^I = V \otimes D_2 + U \otimes D_4^I, \quad (10)$$

where V is the $2 \rightarrow 4$ gluon vertex. It consists of three pieces and it can be expressed in terms of one single function:

$$V = V(1,2,3,4) + V(1,3,2,4) + V(1,4,2,3). \quad (11)$$

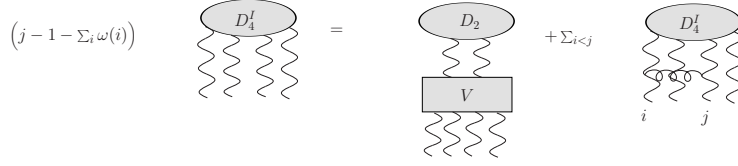


Figure 4: Defining equation for D_4^I .

When projecting on color singlet states in the pairs (1,2) and (3,4), in the limit $N_c \rightarrow \infty$ the first (planar) term represents the leading part, the remaining two terms are nonplanar and subleading in $1/N_c$. Note that this vertex V can be obtained from D_4 only after the removal of the reggeizing pieces D_4^R . As a result of this ‘reduction’ procedure, the vertex V contains both ‘real’ and ‘virtual’ contributions, very much in the same way as the full BFKL kernel has both real and virtual pieces: the latter ones belong to the reggeization of the gluon. In this sense the connection between V and real gluon production is more complicated than in the case of Z .

Now let us rewrite the equation (10). Insert $D_4^I = D_4 - D_4^R$ and regroup the terms:

$$S_4 D_4 = S_4 D_4^R - U \otimes D_4^R + V \otimes D_2 + U \otimes D_4. \quad (12)$$

Making use of the color projector P and taking the limit $N_c \rightarrow \infty$ (with help of the identity (6)) we find:

$$S_4 D_4^\infty = (P S_4 - P U_0 \otimes) D_4^R + P V \otimes D_2 + P U_0 \otimes D_4^\infty. \quad (13)$$

Here the large- N_c limit of PV selects the planar part of the $2 \rightarrow 4$ gluon vertex. Comparison of (13) with (8) leads to

$$Z \otimes D_2 = P \left(V \otimes D_2 + (S_4 - U_0 \otimes) D_4^R - D_{40} \right). \quad (14)$$

This equation establishes the relation between the two versions of the triple Pomeron vertex. The explicit expression for Z was first obtained in [8] and its relation with V derived in [9, 12].

The important point is that the original form (8) has a very transparent physical interpretation. The solution of (8) is a sum of two terms, each with two pomeron legs attached to them. The first, coming from D_{40} , has the meaning of double pomeron exchange with only $q\bar{q}$ intermediate states (low mass diffraction). The second, involving Z , corresponds to high-mass diffraction, with many gluons in the intermediate states. So one would expect that the inclusive cross-section will be given in terms of this vertex Z coupled to two pomerons at rapidity y . This expectation is confirmed [2] by the calculation of the diffractive cross-sections at $t = 0$ in the lowest (non-trivial) order, i.e. of the cross section for diffractive $q\bar{q}$ production. Below we will show that the expression for Z coincides with the diffractive vertex in [2].

Finally note that in all our discussion we have kept rapidity y fixed. If one is interested in cross-sections integrated over the rapidities of observed jets, it is more convenient to use the total amplitude in the form (9), i.e. to write the cross-section as a sum of the reggeized zero-order term directly coupled to the initial and final projectiles plus the contribution from the triple pomeron coupling with the vertex V .

2.2 The explicit form of the diffractive triple Pomeron vertex

In this subsection we use Eq. (14) and derive an explicit expression for the diffractive vertex Z . As found in [1, 2], the explicit form of D_4^R is given by

$$D_4^R = C \left\{ \sum_{i=1}^4 D_2(i) - D_2(12) - D_2(13) - D_2(14) \right\}. \quad (15)$$

(here the argument ‘12’ is a shorthand notation for the sum of the momenta $k_1 + k_2$). For the momenta we will assume that $k_1 + k_2 = -k_3 - k_4 = q$; in short: $12 = -34 = q$. The coefficient $C = (1/2)g^2$ will be suppressed in the following for brevity and has to be re-added in front of our final expressions. In the following (and in a similar derivation in Sec. 3, Eqs. (59)-(64)) we assume that the colour projector P has been applied to operators acting on final gluons, which we do not specify explicitly.

At its lower end the vertex $Z \otimes D_2$ couples to two color singlet pairs, each being symmetric in its transverse momenta, and the whole expression has to be symmetric under the interchange of the two pomerons. So the quantity we have to study,

$$X = \left((j-1) - \sum_{i=1}^4 \omega(i) - U_0 \right) D_4^R - D_{40}, \quad (16)$$

is to be convoluted with a function which is symmetric in the momenta (12) and (34), and also under the interchange (12) \leftrightarrow (34). Since the operator $S_4 - PU_0$ also possesses this symmetry, it follows that for our purpose it would be sufficient to take

$$D_4^R = 4D_2(1) - D_2(12) - 2D_2(13). \quad (17)$$

Nevertheless, we shall consider the general form valid for the a generic momentum configuration (1, 2, 3, 4) of the four t -channel gluons, and we impose the restriction to the diffractive case $12 = -34 = q$ only at the end of our calculation.

Now we evaluate eq. (16). The simplest term is the first one: we have to apply the operator $(S_4 - U_0)$ to the expression (15). We write the nonforward inhomogeneous BFKL equation (2) in a slightly different form:

$$\begin{aligned} (j-1)D_2(i) &= D_{20}(i) + (V_{12} + \omega(1) + \omega(2)) \otimes D_2 \\ &= (j-1)D_2(i) = D_{20}(i) + G(i, 0, 1234 - i), \end{aligned} \quad (18)$$

where $G(1, 2, 3)$ is the infrared finite and conformally invariant function introduced in [2, 9, 12]. It has the form

$$G(1, 2, 3) = G(3, 2, 1) = -W(1, 2, 3) - D_2(1)[\omega(2) - \omega(23)] - D_2(3)[\omega(2) - \omega(12)], \quad (19)$$

where

$$W(1, 2, 3) = \int \frac{d^2 k'_1}{(2\pi)^3} K_{2 \rightarrow 3}(1, 2, 3 | 1', 3') D_2(1'), \quad (20)$$

$$K_{2 \rightarrow 3}(1, 2, 3; 1', 3') = V_{13}(2, 3; 1 - 1', 3') - V_{12}(12, 3; 1', 3'), \quad (21)$$

and on the rhs of the last equation we have written explicitly the incoming and outgoing momenta of the BFKL kernel V_{ij} . Taking into account (15) we find

$$(j-1)D_4^R = D_{40} + \sum_{i=1}^4 G(i, 0, 1234 - i) - G(12, 0, 34) - G(13, 0, 24) - G(14, 0, 23). \quad (22)$$

Continuing with the rhs of Eq. (16), we postpone the terms $-\sum_{i=1}^4 \omega(i)D_4^R$: later on we shall see that they will cancel with similar terms coming from the other parts of the operator. So we have to study only the terms with U_0 which we will denote by

$$\tilde{X} = -U_0 \otimes D_4^R. \quad (23)$$

The final X will be given by

$$X = \sum_{i=1}^4 G(i, 0, 1234 - i) - G(12, 0, 34) - G(13, 0, 24) - G(14, 0, 23) - \sum_{i=1}^4 \omega(i)D_4^R + \tilde{X}. \quad (24)$$

For the further investigation of X we shall make use of two identities derived earlier:

1) the bootstrap relation [13]:

$$V_{23}D_2(1) = 2[\omega(23) - \omega(2) - \omega(3)]D_2(1); \quad (25)$$

2) the action of an interaction on D_2 with a shifted argument [8]:

$$V_{12}D_2(13) = W(24, 1, 3) + W(4, 2, 13) - W(3, 12, 4) - W(13, 0, 24). \quad (26)$$

Here the function W has been defined in (20). A special case of (20) is

$$V_{13}(1, 3; 1', 3') \otimes D_2(1) = -W(1, 0, 3). \quad (27)$$

Armed with these identities we may start calculating various terms in X , similarly to [9, 12]. We begin with

$$\tilde{X}_1 = -U_0 \otimes D_2(1) = -(V_{12} + V_{34}) \otimes D_2(1). \quad (28)$$

The term independent of 1 will give :

$$-V_{34} \otimes D_2(1) = -2[\omega(34) - \omega(3) - \omega(4)]D_2(1). \quad (29)$$

The term with V_{12} leads to

$$V_{12} \otimes D_2(1) = W(1, 2, 34) - W(1, 0, 234). \quad (30)$$

Summing the two terms leads to

$$\tilde{X}_1 = [W(1, 0, 234) - W(1, 2, 34)] - 2[\omega(34) - \omega(3) - \omega(4)]D_2(1). \quad (31)$$

An analogous result with just a simple permutation of the momentum labels is easily obtained for the terms $-U_0 \otimes D_2(2)$, $-U_0 \otimes D_2(3)$, and $-U_0 \otimes D_2(4)$. Now we pass to the second term in (17) and compute

$$\tilde{X}_{12} = -U_0 \otimes D_2(12) = -(V_{12} + V_{34}) \otimes D_2(12). \quad (32)$$

Here, due to the bootstrap relation:

$$\tilde{X}_{12} = -2[\omega(12) - \omega(1) - \omega(2) + \omega(34) - \omega(3) - \omega(4)]D_2(12), \quad (33)$$

both terms can be expressed in terms of trajectory functions. From the third term in (17) we have

$$\tilde{X}_{13} = -U_0 \otimes D_2(13) = -(V_{12} + V_{34}) \otimes D_2(13). \quad (34)$$

Here both terms are non-trivial:

$$V_{12} \otimes D_2(13) = W(24, 1, 3) + W(4, 2, 13) - W(4, 12, 3) - W(24, 0, 13), \quad (35)$$

$$V_{34} \otimes D_2(13) = W(13, 4, 2) + W(1, 3, 24) - W(1, 34, 2) - W(24, 0, 13). \quad (36)$$

In the sum we obtain:

$$\begin{aligned} \tilde{X}_{13} = & [2W(13, 0, 24) + W(1, 34, 2) + W(3, 12, 4) \\ & - W(1, 34, 2) - W(2, 4, 13) - W(3, 1, 24) - W(4, 2, 13)]. \end{aligned} \quad (37)$$

In our next step we rewrite these expressions in terms of the G function. This is lengthy but straightforward. By definition

$$G(1, 2, 3) = G(3, 2, 1) = -W(1, 2, 3) - D_2(1)[\omega(2) - \omega(23)] - D_2(3)[\omega(2) - \omega(12)], \quad (38)$$

from which we arrive at:

$$W(1, 2, 3) = -G(1, 2, 3) - D_2(1)[\omega(2) - \omega(23)] - D_2(3)[\omega(2) - \omega(12)]. \quad (39)$$

So all we have to do is to substitute this expression for W into our formulas for the different \tilde{X} . This substitution obviously expresses W through G functions and through terms with ω 's. These additional ω terms can be combined into a simple combination (Appendix 1) which, when combined with similar terms in eq. (24), cancels.

As a result, summing all terms as indicated in (16) we get

$$\begin{aligned} \tilde{X} = & -U_0 \otimes D_4^R = G(1, 2, 34) + G(2, 1, 34) + G(12, 3, 4) + G(12, 4, 3) + 2G(1, 34, 2) + 2G(3, 12, 4) \\ & - G(1, 4, 23) - G(4, 1, 23) - G(14, 2, 3) - G(14, 3, 2) - G(1, 3, 24) - G(3, 1, 24) \\ & - G(2, 4, 13) - G(4, 2, 13) + 2G(13, 0, 24) + 2G(14, 0, 23) \\ & - G(1, 0, 234) - G(2, 0, 341) - G(3, 0, 412) - G(4, 0, 123) + \sum_{i=1}^4 \omega(i) D_4^R. \end{aligned} \quad (40)$$

Inserting this into (24), the terms proportional to $\sum_{i=1}^4 \omega(i) D_4^R$ cancel, and X can be expressed entirely in terms of G functions:

$$\begin{aligned} X = & G(1, 2, 34) + G(2, 1, 34) + G(12, 3, 4) + G(12, 4, 3) + 2G(1, 34, 2) + 2G(3, 12, 4) \\ & - G(1, 4, 23) - G(4, 1, 23) - G(14, 2, 3) - G(14, 3, 2) - G(1, 3, 24) - G(3, 1, 24) \\ & - G(2, 4, 13) - G(4, 2, 13) + G(13, 0, 24) + G(14, 0, 23) - G(12, 0, 34). \end{aligned} \quad (41)$$

Returning to the lhs of Eq. (14), we express V in terms of G 's [2]:

$$\begin{aligned} V(1, 2, 3, 4) = \lim_{N_c \rightarrow \infty} PV \otimes D_2 = & G(1, 23, 4) + G(2, 13, 4) + G(1, 24, 3) + G(2, 14, 3) - G(12, 3, 4) \\ & - G(12, 4, 3) - G(1, 2, 34) - G(2, 1, 34) + G(12, 0, 34). \end{aligned} \quad (42)$$

Using Eq.(14) for Z and Eq. (41) leads to our final expression:

$$\begin{aligned} Z(1, 2, 3, 4) \equiv Z \otimes D_2 = & G(1, 23, 4) + G(2, 14, 3) + G(1, 24, 3) + G(2, 13, 4) \\ & - G(13, 2, 4) - G(13, 4, 2) - G(1, 3, 24) - G(3, 1, 24) \\ & - G(14, 2, 3) - G(14, 3, 2) - G(1, 4, 23) - G(4, 1, 23) \\ & + 2G(1, 34, 2) + 2G(3, 12, 4) + G(13, 0, 34) + G(14, 0, 23), \end{aligned} \quad (43)$$

which is the result firstly derived in [9, 12], where it was already observed that Z can also be expressed in terms of the nonplanar pieces of the V function (cf. Eq. (11) in [2])

$$Z(1, 2, 3, 4) = V(1, 3, 2, 4) + V(1, 4, 2, 3). \quad (44)$$

Let us finally compare our result with the diffractive vertex derived in [2]. We consider diffractive kinematics, i.e. we put the total transverse momentum equal to zero and we assign the momenta:

$$1 = l, \quad 2 = -l, \quad 3 = m, \quad 4 = -m, \quad (t = 0).$$

In this configuration we find, using the notation of [2] for the G function which depends only on two arguments:

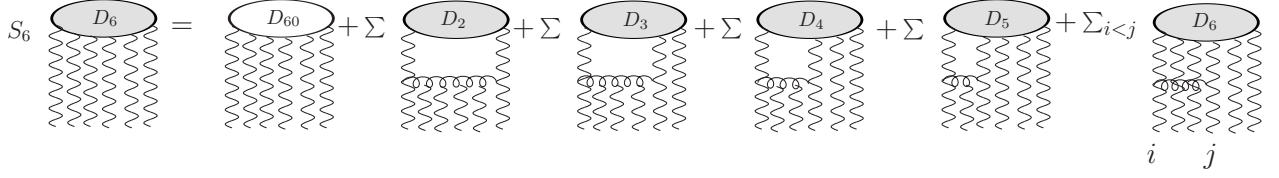
$$\begin{aligned} Z \otimes D_2 = & 2G(l, -l) + 2G(m, -m) + G(l, -m) + 2G(l, m) + G(l + m, -l, -m) + G(l - m, m - l) \\ & - 2G(l, m - l) - 2G(-l, m + l) - 2G(m, l - m) - 2G(-m, l + m). \end{aligned} \quad (45)$$

This coincides with the expression for the diffractive cross-section at $t = 0$ in [2]; in contrast to the triple Pomeron vertex V which also contains virtual contributions this diffractive triple Pomeron contains only the real contribution for produced gluons. Note that the diffractive vertex in [2] has been derived without any large- N_c approximation.

Let us comment on our result. Equation (8) (Fig. 3b) which defines the diffractive vertex Z contains, as an inhomogeneous term, the coupling of two color singlet pairs of gluons to the $q\bar{q}$ system: solving the equation by iteration generates the two Pomeron ladders which are needed to derive a cross section formula for the diffractive production of a $q\bar{q}$ pair (double Pomeron exchange). This to be compared with equation (10) (Fig. 4) for D_4^I where such a term is absent; also D_4^R , which is a sum of BFKL ladders D_2 , does not have such a term. On the other hand, the function D_4 from which both D_4^I and D_4^R are obtained contains this contribution. The explanation is the following: while splitting D_4 into the sum $D_4^R + D_4^I$ we regroup the QCD diagrams in a special way; as a consequence of the bootstrap, the two-Pomeron ladder diagrams are absorbed into D_4^R and into $V \otimes D_{20}$. In the latter piece, the double Pomeron exchange becomes part of the virtual contributions inside the triple Pomeron vertex V .

A second comment applies to the transformation to configuration space. Whereas the momentum space representation of high energy QCD allows a direct control of the s -channel unitarity content of scattering amplitudes, in recent years an appealing intuitive picture of high energy scattering processes has been developed in configuration space, the QCD dipole picture [14]. A popular model is the nonlinear Balitsky-Kovchegov (BK) equation [3], whose structure is reminiscent of the fan diagram equation of [15]. In a recent paper [5] the BK equation has been compared with the nonlinear equation which sums fan diagrams built from BFKL ladders. Making use of symmetry properties of the BFKL kernel, it has been shown, for a particular set of initial conditions, that in the large- N_c limit the kernel of the BK-equation coincides with the Fourier transform of the triple Pomeron vertex V . From this one concludes that the BK equation 'knows' about the two-Pomeron ladder diagrams, but it contains only a part of them.

Conversely, one might want to know the Fourier transform of the diffractive triple Pomeron vertex Z , eq.(44). As shown in [5] the first nonleading (in $1/N_c^2$) corrections to the fan diagram equation are due to the nonplanar part of the triple Pomeron vertex V which coincides with the rhs of Eq. (44), $V(1, 3, 2, 4) + V(1, 4, 2, 3)$. Its Fourier transform turns out to be simple, but quite different from the BK kernel. This, once more, confirms that the BK kernel contains a part of the two Pomeron ladder diagrams: the diffractive vertex Z , by definition, excludes these contributions, and its Fourier transform is different from the BK kernel. As a conclusion, a full account of the $q\bar{q}$ diffractive intermediate state cannot be obtained by

Figure 5: Equation for D_6 .

simply adding this contribution to the BK equation; this would lead to an overcounting of QCD diagrams.

3 Diffractive vertex for the splitting $P \rightarrow OO$

3.1 Definitions

We now turn to six gluons and repeat the same line of arguments. We start from the integral equation for D_6 [10] which sums all gluon production diagrams (Fig. 5). It has the structure

$$S_6 D_6 = D_{60} + \sum K_{2 \rightarrow 6} \otimes D_2 + \sum K_{2 \rightarrow 5} \otimes D_3 + \sum K_{2 \rightarrow 4} \otimes D_4 + \sum K_{2 \rightarrow 3} \otimes D_5 + U \otimes D_6, \quad (46)$$

where

$$U = \sum_{(i,j)} V_{ij} \quad (47)$$

denotes the sum over all pairwise interactions, and $S_6 = j - 1 - \sum_{i=1}^6 \omega(i)$ is the natural extension of S_4 for the 6 reggeized gluon propagator. Invoking the bootstrap properties of D_3 and D_5 and the decomposition (9) for D_4 , one immediately sees that, on the rhs of (46), the second and the third term, the D_4^R pieces in the fourth and the fifth term all start with a two-gluon states below the quark loop and then define an effective $2 \rightarrow 6$ vertex, \tilde{Z} . Let P be a projector on the two Odderon state (Odderons are formed by the triplets (123) and (456)), and let P act on both sides of (46). Next we take the large N_c limit: define $D_6^\infty = \lim_{N_c \rightarrow \infty} P D_6$ and

$$U_0 = V_{12} + V_{23} + V_{31} + V_{45} + V_{56} + V_{64}, \quad (48)$$

Then we have, in analogy with (6):

$$P U \otimes D_6^\infty = P U_0 \otimes P D_6 = P U_0 \otimes D_6^\infty. \quad (49)$$

In [16] it has been shown that, on the rhs of (46), the D_4^I pieces in the fourth and the fifth term contain transitions of a two-Pomeron state to a two-Odderon state which, at large N_c , are suppressed. As a result, in the large- N_c limit Eq. (46) takes the simple form

$$S_6 D_6^\infty = P D_{60} + Z \otimes D_2 + P U_0 \otimes D_6^\infty, \quad (50)$$

where $Z = \lim_{N_c \rightarrow \infty} P \tilde{Z}$. This equation defines the diffractive POO vertex, Z .

The solution to this equation has a transparent physical meaning. It is given by attaching two odderon Green functions (formed by gluons (123) and (456)) to the two inhomogeneous terms. In the first term, the Odderons couple directly to the $q\bar{q}$ pair (double Odderon exchange), in the second term the Odderons couple to the vertex Z which describes diffractive

gluon production. After convoluting the Odderon Green's functions with appropriate impact factors (e.g. couplings to the proton), one obtains cross sections for 'diffraction with Odderon exchange'.

To find the connection between the diffractive vertex Z and the POO vertex W defined in [10], we proceed in the same way as in the previous section. We start from the decomposition of D_6 into symmetric and antisymmetric pieces, i.e. we invoke the reduction procedure which leads to

$$D_6 = D_6^R + D_6^I. \quad (51)$$

In fact, as discussed in [10], D_6^I is not yet fully reduced and requires a further step of reduction. However, for our purpose of discussing the splitting into two odderon configurations, this is second step not necessary. The non-reggeizing ("irreducible") piece D_6^I satisfies the equation

$$S_6 D_6^I = W \otimes D_2 + \sum L + \sum I + \sum J + \sum K_{2 \rightarrow 4} \otimes D_4^I + \sum K_{2 \rightarrow 3} \otimes D_5^I + \sum U \otimes D_6^I. \quad (52)$$

which defines the POO vertex W . Projecting with P onto the 2-Odderon state, and taking the large- N_c limit, we use the results of [16]: only the first and the last term on the rhs of (52) survive. Substituting the relation $D_6^I = D_6 - D_6^R$, we obtain

$$S_6 P(D_6 - D_6^R) = PW \otimes D_2 + PU \otimes (D_6 - D_6^R), \quad (53)$$

which leads to

$$S_6 D_6^\infty = (S_6 - PU_0 \otimes) D_6^R + PW \otimes D_2 + PU_0 \otimes D_6^\infty. \quad (54)$$

Comparison with eq. (50) gives the relations between the two vertices:

$$PW \otimes D_2 = PD_{60} + Z \otimes D_2 - P(S_6 - U_0 \otimes) D_6^R \quad (55)$$

and

$$Z \otimes D_2 = P \left(W \otimes D_2 + (S_6 - U_0 \otimes) D_6^R - D_{60} \right). \quad (56)$$

3.2 The explicit form of the diffractive POO vertex

We now turn to the explicit form of Z . As shown in [10], the explicit form of D_6^R in the two-odderon colour channel is given by

$$\begin{aligned} D_6^R = C \Big\{ & \sum_{i=1}^6 D_2(i) - \sum_{i \neq k=1}^3 D_2(ik) \\ & - \sum_{i \neq k=4}^6 D_2(ik) - \sum_{i=1}^3 \sum_{k=4}^6 D_2(ik) + \sum_{i \neq k=1}^3 \sum_{l=4}^6 D_2(ikl) + D_2(123) \Big\}, \end{aligned} \quad (57)$$

where the odderons are assumed to carry total momenta $123 = -456 = q$. The coefficient C already contains the color projection and has the form:

$$C = g^4 d^{abcdef} d_{abc} d_{def} = g^4 \frac{(N_c^2 - 1)^2 (N_c^2 - 4)^2}{8N_c^3}. \quad (58)$$

As in the previous section, in the following we will suppress this coefficient, but keep in mind that, at the end, it has to be re-added in front of our final expressions. The vertex $Z \otimes D_2$ can be viewed as an operator which acts on the two odderons. Each odderon state is symmetric

in its three momentum variables, and the whole vertex is symmetric under the interchange of the two odderons. So the quantity we have to study

$$X = (S_6 - U_0)D_6^R - D_{60} \quad (59)$$

is to be convoluted with functions which are symmetric in the momenta (123) and (456), and also symmetric under the interchange (123) \leftrightarrow 456. Since the operator $S_6 - U_0$ also possesses this symmetry, it follows that all terms inside each of the sums in (57) will give equal contributions, that is, for our purpose we may simply write

$$D_6^R = 6D_2(1) - 6D_2(12) - 9D_2(14) + 9D_2(124) + D_2(123). \quad (60)$$

Different terms in the operator $(S_6 - U_0)$ are again to be treated in a different manner. The simplest term with the factor $j - 1$ according to (2) leads to

$$(j-1)D_6^R = D_{60} + 6G(1, 0, -1) - 6G(12, 0, -12) - 9G(14, 0, -14) \\ + 9G(124, 0, -124) + G(123, 0, -123). \quad (61)$$

Terms from $-\sum_{i=1}^6 \omega(i)$ will cancel with similar terms coming from the remaining part of the operator. So, in fact, we have to study only $U_0 D_6^R$. On defining

$$\tilde{X} = -2U_0 D_6^R, \quad (62)$$

the final X will be given by

$$X = \frac{1}{2} \left[2 \left(j - 1 - \sum_{i=1}^6 \omega(i) \right) D_6^R + \tilde{X} \right] - D_{60}. \quad (63)$$

Using the identities (25) and (26) one can express, in (62), all terms via the trajectory function $\omega(i)$ and the function W . Subsequently one reexpresses functions W via the G 's and verifies that all terms proportional to ω 's cancel. The procedure closely follows the one for the triple pomeron vertex presented in the preceding section, but it requires considerably more algebra. We present it in the Appendix 2. Our final expression for X in terms of G 's is

$$X = 9G(14, 0, -14) - 9G(124, 0, -124) + G(123, 0, -123) - 6G(12, 3, -123) + 6G(1, 23, -123) \\ - 18G(4, 1, -14) + 18G(4, 12, -124) + 18G(12, 4, -124) - 18G(14, 23, 56). \quad (64)$$

3.3 The symmetry structure of X and the final diffractive vertex

The vertex W in [10] has a peculiar symmetry structure [17, 16]. It can be made explicit by introducing operators \hat{S} and \hat{P} which act on antisymmetric functions in the following way:

$$\hat{S}\phi(1, 2) = \frac{1}{2} \sum_{(123)} [\phi(12, 3) - \phi(3, 12)], \quad (65)$$

where the sum is taken over the cyclic permutations of (123), and

$$\hat{P}\phi(1, 2) = \frac{1}{2} [\phi(123, 0) - \phi(0, 123)]. \quad (66)$$

Then, ignoring the overall coefficient C (Eq. (58)), the symmetric vertex W can be cast into the form

$$W(1, 2, 3|4, 5, 6) = -4(\hat{S}_1 - \hat{P}_1)f(1, 2|4, 5)(\hat{S}_2^\dagger - \hat{P}_2^\dagger). \quad (67)$$

Here

$$f(1, 2|4, 5) = \frac{1}{4}[G(1, 24, 5) - G(2, 14, 5) - G(1, 25, 4) + G(2, 15, 4)] \equiv \mathcal{A}G(1, 24, 5). \quad (68)$$

The operator \mathcal{A} antisymmetrizes in each pair of variables 12 and 34, whereas the operators \hat{S}_1, \hat{P}_1 act on the variables 1 and 2, and the operators \hat{S}_2, \hat{P}_2 on the variables 4 and 5.

One may ask if also the contribution X to the vertex Z derived above can be cast into a similar form, with the function f being replaced by a different function g which again can be expressed in terms of G 's. Obviously, apart from (68), we can have only two antisymmetric functions

$$g^{(1)} = \mathcal{A}G(14, 2, 5), \quad \text{and} \quad g^{(2)} = \mathcal{A}G(14, 5, 2). \quad (69)$$

Due to symmetry $(123) \leftrightarrow (456)$, after integration with two odderon Green's functions, they will give the same result. So it is sufficient to study

$$U_0^{(1)} = (\hat{S}_1 - \hat{P}_1)g^{(1)}(\hat{S}_2^\dagger - \hat{P}_2^\dagger). \quad (70)$$

Performing the necessary operations and using the symmetries we find

$$\begin{aligned} 4U_0^{(1)} &= 9G(4, 1, -14) - 9G(4, 12, -124) - 9G(12, 4, 124) + 9G(14, 23, 56) - 3G(1, 0, -1) \\ &\quad + 3G(12, 0, -12) + 3G(12, 3, -123) - 3G(1, 23, -123) - G(123, 0, 456). \end{aligned} \quad (71)$$

Comparing with our expression for X we find

$$\begin{aligned} X &= -8U_0^{(1)} - 6G(1, 0, -1) + 6G(12, 0, -12) + 9G(14, 0, -14) \\ &\quad - 9G(124, 0, -124) - G(123, 0, -123) \\ &= -8U_0^{(1)} + H_2 D_6^R, \end{aligned} \quad (72)$$

where $H_2(1, 2) = -\omega(1) - \omega(2) - V_{12}$ is the BFKL "Hamiltonian". This allows to write the complete diffractive vertex Z in the form

$$\begin{aligned} Z(1, 2, 3|4, 5, 6) &= \\ &-4(\hat{S}_1 - \hat{P}_1)\mathcal{A}\left(G(1, 24, 5) + G(14, 2, 5) + G(14, 5, 2)\right)(\hat{S}_2^\dagger - \hat{P}_2^\dagger) + H_2 D_6^R(1, 2, 3|4, 5, 6). \end{aligned} \quad (73)$$

Thus it almost matches the structure of the symmetric vertex, except for the extra term, which has quite a transparent meaning. Asymptotically, at high rapidities,

$$H_2 D_6^R = -\Delta D_6^R, \quad (74)$$

where Δ is the BFKL intercept.

The obtained diffractive vertex is infrared finite by itself (and conformally invariant), since it is a sum of infrared finite and conformally invariant functions G . One may ask if the infrared stability is preserved when this vertex is coupled to the two odderons, which involves an integration over all six momenta 1,2,...6 with a denominator $\prod_{i=1}^6 k_i^2$. The answer is affirmative, since, in fact, the odderons are better behaved at $k_i^2 \rightarrow 0$ than one might deduce from these denominators. Let us take, as an example, the odderon constructed in [17]. It has a structure

$$O(1, 2, 3) = \frac{1}{k_1^2 k_2^2 k_3^2} \left(P_a(12, 3) + P_a(23, 1) + P_a(31, 2) \right),$$

where $P_a(1, 2)$ is an amputated antisymmetric pomeron state. Consider the limit $k_1 \rightarrow 0$. The bracket becomes

$$P_a(2, 3) + P_a(23, 0) + P_a(3, 2),$$

which is zero, since P_a is antisymmetric and goes to zero as one of its arguments goes to zero. So the pole singularity at $k_1^2 \rightarrow 0$ is softened. As a result, the intergration of k_1 and of the other momenta will not encounter any difficulties in the infrared region.

4 Conclusions

In this paper we have discussed a particular aspect of both the triple Pomeron vertex and the Pomeron→Odderon-Odderon vertex in pQCD. For the calculation of diffractive cross sections, it is useful to define vertices which differ from those which appear in inclusive cross sections or in the context of QCD reggeon field theory. We have named them ‘diffractive vertex’. As to the triple Pomeron case, we have reviewed and analysed the relation between this diffractive vertex and the standard triple Pomeron vertex. This analysis has allowed to draw conclusions for the s -channel content of the kernel of the BK equation. We then have applied the same line of arguments to the Pomeron→Odderon-Odderon vertex, and we have derived the new diffractive vertex to be used in the computation of diffractive states with Odderon exchange. For simplicity, we have restricted ourselves to the large- N_c limit.

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6 Appendix 1. Terms with ω 's in the vertex PPP

To check the cancellation of all the terms with ω 's we now calculate the last term which adds to the G 's in the expression (40). We write those in \tilde{X} as Y , and in particular Y_a is related to the \tilde{X}_a term, where $a = 1, 2, 3, 4, 12, 13, 14$ is any string of the momenta present in the D_4^R expression.

Start with the term \tilde{X}_1 :

$$Y_1 = D_2(1)[-2\omega(34) - 2\omega(3) - 2\omega(4) + \omega(2) + \omega(1)] + D_2(12)[\omega(2) - \omega(12)]. \quad (75)$$

An analogous result is obtained for the other the cases \tilde{X}_2 , \tilde{X}_3 and \tilde{X}_4 . From \tilde{X}_{12} :

$$Y_{12} = D_2(12)[2 \sum_{i=1}^4 \omega(i) - 2\omega(12) - 2\omega(34)]. \quad (76)$$

There are quite a few terms from \tilde{X}_{13} . They sum into

$$\begin{aligned} Y_{13} = & D_2(13) \sum_{i=1}^4 \omega(i) + D_2(1)[\omega(3) - \omega(34)] + D_2(2)[\omega(4) - \omega(34)] \\ & + D_2(3)[\omega(1) - \omega(12)] + D_2(4)[\omega(2) - \omega(12)]. \end{aligned} \quad (77)$$

A similar result is obtained for Y_{14} . Now we consider the expression

$$Y = \sum_{i=1}^4 Y_i - Y_{12} - Y_{13} - Y_{14} = \sum_a c_a D_2(a). \quad (78)$$

The coefficient of the $D_2(1)$ term in Y is given by

$$c_1 = -2\omega(34) + 2\omega(3) + 2\omega(4) + \omega(2) + \omega(1) - \omega(4) + \omega(34) - \omega(3) + \omega(34) = \sum_{i=1}^4 \omega(i) \quad (79)$$

and similarly one gets $c_2 = c_3 = c_4 = c_1$. We consider now c_{12} :

$$\begin{aligned} c_{12} &= 2\omega(12) + 2\omega(34) - 2 \sum_{i=1}^4 \omega(i) + \omega(2) - \omega(12) + \omega(1) - \omega(12) \\ &\quad + \omega(4) - \omega(34) + \omega(3) - \omega(34) = - \sum_{i=1}^4 \omega(i). \end{aligned} \quad (80)$$

Finally let us consider c_{13} , which is immediately seen to be:

$$c_{13} = - \sum_{i=1}^4 \omega(i), \quad (81)$$

and the same result holds for c_{14} . This means we have proved that

$$Y = \sum_{i=1}^4 \omega(i) D_4^R, \quad (82)$$

which is the desired result which allows the cancellation in X .

7 Appendix 2. Useful formulas for the POO diffractive vertex

7.1 Action of U_0 on different terms in D_6^R

We begin with

$$\tilde{X}_1 = -2U_0 D_2(1) = -(V_{12} + V_{23} + V_{31} + V_{45} + V_{56} + V_{64}) D_2(1). \quad (83)$$

Terms independent of 1 will give ω 's:

$$\begin{aligned} &-(V_{23} + V_{45} + V_{56} + V_{64}) D_2(1) \\ &= -2[\omega(23) - \omega(2) - \omega(3) + \omega(45) - \omega(4) - \omega(5) + \omega(56) - \omega(5) - \omega(6) + \omega(64) - \omega(6) - \omega(4)]. \end{aligned} \quad (84)$$

We are left with terms which contain

$$(V_{12} + V_{13}) D_2(1) = W(1, 2, -12) + W(1, 3, -13) - 2W(1, 0, -1). \quad (85)$$

Summing all terms we get

$$\begin{aligned} \tilde{X}_1 &= [2W(1, 0, -1) - W(1, 2, -12) - W(1, 3, -13)] \\ &\quad - 2[\omega(23) - \omega(2) - \omega(3) + \omega(45) - \omega(4) - \omega(5) \\ &\quad + \omega(56) - \omega(5) - \omega(6) + \omega(64) - \omega(6) - \omega(4)] D_2(1) \\ &= [2W(1, 0, -1) - W(1, 2, -12) - W(1, 3, -13)] + \\ &\quad [2 \sum_{i=2}^3 \omega(i) + 4 \sum_{i=4}^6 \omega(i) - 2\omega(23) - 2\omega(45) - 2\omega(56) - 2\omega(64)] D_2(1). \end{aligned} \quad (86)$$

Now we pass to the second term in (60):

$$\tilde{X}_{12} = -2U D_2(12) = -(V_{12} + V_{23} + V_{31} + V_{45} + V_{56} + V_{64}) D_2(12). \quad (87)$$

Terms which give ω 's are:

$$\begin{aligned} & -(V_{12} + V_{45} + V_{56} + V_{64})D_2(12) \\ & = -2[\omega(12) - \omega(1) - \omega(2) + \omega(45) - \omega(4) - \omega(5) + \omega(56) - \omega(5) - \omega(6) + \omega(64) - \omega(6) - \omega(4)]. \end{aligned} \quad (88)$$

We are left with two terms which contain

$$V_{13}D_2(12) = W(-12, 1, 2) + W(-123, 3, 12) - W(2, 13, -123) - W(12, 0, -12) \quad (89)$$

and

$$V_{23}D(12) = W(-12, 2, 1) + W(-123, 3, 12) - W(1, 23, -123) - W(12, 0, -12). \quad (90)$$

Summing all the terms we get

$$\begin{aligned} \tilde{X}_{12} &= -[W(-12, 1, 2) + W(-123, 3, 12) - W(2, 13, -123) \\ & - W(12, 0, -12) + W(-12, 2, 1) + W(-123, 3, 12) - W(1, 23, -123) - W(12, 0, -12)] \\ & - 2[\omega(12) - \omega(1) - \omega(2) + \omega(45) - \omega(4) - \omega(5) \\ & + \omega(56) - \omega(5) - \omega(6) + \omega(64) - \omega(6) - \omega(4)]D_2(12) = \\ & = +[2W(12, 0, -12) - W(-12, 1, 2) \\ & - 2W(-123, 3, 12) + W(2, 13, -123) - W(-12, 2, 1) + W(1, 23, -123)] \\ & + [2\omega(1) + 2\omega(2) + 4 \sum_{i=4}^6 \omega(i) - 2\omega(12) - 2\omega(45) - 2\omega(56) - 2\omega(64)]D_2(12). \end{aligned} \quad (91)$$

From the third term in (60) we have

$$\tilde{X}_{14} = -(V_{12} + V_{23} + V_{31} + V_{45} + V_{56} + V_{64})D_2(14). \quad (92)$$

Terms which give ω 's are:

$$-(V_{23} + V_{56})D_2(12) = -2[\omega(23) - \omega(2) - \omega(3) + \omega(56) - \omega(5) - \omega(6)]D_2(14). \quad (93)$$

Now we have 4 non-trivial terms containing

$$V_{12}D_2(14) = W(-14, 1, 4) + W(-124, 2, 14) - W(4, 12, -124) - W(14, 0, -14), \quad (94)$$

$$V_{13}D(14) = W(-14, 1, 4) + W(-134, 3, 14) - W(4, 13, -134) - W(14, 0, -14), \quad (95)$$

$$V_{45}D(14) = W(-14, 4, 1) + W(-145, 5, 14) - W(1, 45, -145) - W(14, 0, -14), \quad (96)$$

$$V_{46}D(14) = W(-14, 4, 1) + W(-146, 6, 14) - W(1, 46, -146) - W(14, 0, -14). \quad (97)$$

Summing all:

$$\begin{aligned} \tilde{X}_{14} &= [-W(-14, 1, 4) - W(-124, 2, 14) + W(4, 12, -124) \\ & + W(14, 0, -14) - W(-14, 1, 4) - W(-134, 3, 14) + W(4, 13, -134) + W(14, 0, -14) \\ & - W(-14, 4, 1) - W(-145, 5, 14) + W(1, 45, -145) + W(14, 0, -14) \\ & - W(-14, 4, 1) - W(-146, 6, 14) + W(1, 46, -146) + W(14, 0, -14)] \\ & - 2[\omega(23) - \omega(2) - \omega(3) + \omega(56) - \omega(5) - \omega(6)]D_2(14) \\ & = [4W(14, 0, -14) - 2W(-14, 1, 4) - W(-124, 2, 14) + W(4, 12, -124) \end{aligned}$$

$$\begin{aligned}
& -W(-134, 3, 14) + W(4, 13, -134) \\
& -2W(-14, 4, 1) - W(-145, 5, 14) + W(1, 45, -145) - W(-146, 6, 14) + W(1, 46, -146)] \\
& + [2\omega(2) + 2\omega(3) + 2\omega(5) + 2\omega(6) - 2\omega(23) - 2\omega(56)]D_2(14). \tag{98}
\end{aligned}$$

The fourth term gives contributions like

$$\tilde{X}_{124} = -2UD_2(1) = (V_{12} + V_{23} + V_{31} + V_{45} + V_{56} + V_{64})D_2(124). \tag{99}$$

Terms which give ω 's are:

$$-(V_{12} + V_{56})D_2(124) = -2[\omega(12) - \omega(1) - \omega(2) + \omega(56) - \omega(5) - \omega(6)]D_2(124) \tag{100}$$

and again we have 4 nontrivial terms

$$V_{13}D(124) = W(-124, 1, 24) + W(-1234, 3, 124) - W(24, 13, -1234) - W(124, 0, -124), \tag{101}$$

$$V_{23}D(124) = W(-124, 2, 14) + W(-1234, 3, 124) - W(14, 23, -1234) - W(124, 0, -124), \tag{102}$$

$$V_{45}D(124) = W(-124, 4, 12) + W(-1245, 5, 124) - W(12, 45, -1245) - W(124, 0, -124), \tag{103}$$

$$V_{46}D(124) = W(-124, 4, 12) + W(-1246, 6, 124) - W(12, 46, -1246) - W(124, 0, -124). \tag{104}$$

We sum all to obtain

$$\begin{aligned}
& \tilde{X}_{124} = [-W(-124, 1, 24) - W(-1234, 3, 124) + W(24, 13, -1234) \\
& + W(124, 0, -124) - W(-124, 2, 14) - W(-1234, 3, 124) \\
& + W(14, 23, -1234) + W(124, 0, -124) - W(-124, 4, 12) - W(-1245, 5, 124) \\
& + W(12, 45, -1245) \\
& + W(124, 0, -124) - W(-124, 4, 12) - W(-1246, 6, 124) + W(12, 46, -1246) + W(124, 0, -124)] \\
& - 2[\omega(12) - \omega(1) - \omega(2) + \omega(56) - \omega(5) - \omega(6)]D_2(124) \\
& = [4W(124, 0, -124) - W(-124, 1, 24) - 2W(-1234, 3, 124) + W(24, 13, -1234) \\
& - W(-124, 2, 14) + W(14, 23, -1234) - 2W(-124, 4, 12) \\
& - W(-1245, 5, 124) + W(12, 45, -1245) - W(-1246, 6, 124) + W(12, 46, -1246)] \\
& + [2\omega(1) + 2\omega(2) + 2\omega(5) + 2\omega(6) - 2\omega(12) - 2\omega(56)]D_2(124). \tag{105}
\end{aligned}$$

Finally the last term

$$\tilde{X}_{123} = -2UD_2(123) = (V_{12} + V_{23} + V_{31} + V_{45} + V_{56} + V_{64})D_2(123). \tag{106}$$

Here all terms give ω 's:

$$\begin{aligned}
& -(V(12) + V(23) + V(31) + V(45) + V_{56} + V(64))D_2(124) \\
& = -2[\omega(12) + \omega(23) + \omega(13) + \omega(45) + \omega(56) + \omega(46) - 2\sum_{i=1}^6 \omega(i)]D_2(123). \tag{107}
\end{aligned}$$

So we find

$$\tilde{X}_{123} =$$

$$[4 \sum_{i=1}^6 \omega(i) - 2\omega(12) - 2\omega(23) - 2\omega(31) - 2\omega(45) - 2\omega(56) - 2\omega(46)]D_2(123). \quad (108)$$

Our next step is to transform these expressions to functions G using (39). This changes $g^2 N_c W$ into $-G$ and gives additional terms with ω 's. It is expected that these additional terms with ω 's plus already existing in our expressions will cancel leaving the final expression for the vertex entirely in terms of G . Calculation shows that this is indeed so (see below).

As a result we are left with terms with G 's. Summed as indicated in (60) they give a long expression

$$\begin{aligned} \tilde{X} = & -12G(1, 0, -1) + 6G(1, 2, -12) + 6G(1, 3, -13) + 12G(12, 0, -12) \\ & -6G(-12, 1, 2) - 12G(-123, 3, 12) + 6G(2, 13, -123) - 6G(-12, 2, 1) \\ & + 6G(1, 23, -123) + 36G(14, 0, -14) \\ & -18G(-14, 1, 4) - 9G(-124, 2, 14) + 9G(4, 12, -124) - 9G(-134, 3, 14) + 9G(4, 13, -134) \\ & -18G(-14, 4, 1) - 9G(-145, 5, 14) + 9G(1, 45, -145) - 9G(-146, 6, 14) + 9G(1, 46, -146) \\ & -36G(124, 0, -124) + 9G(-124, 1, 24) + 18G(-1234, 3, 124) \\ & -9G(24, 13, -1234) + 9G(-124, 2, 14) - 9G(14, 23, -1234) \\ & + 18G(-124, 4, 12) + 9G(-1245, 5, 124) - 9G(12, 45, -1245) \\ & + 9G(-1246, 6, 124) - 9G(12, 46, -1246) + [\omega \text{ terms}]. \end{aligned} \quad (109)$$

It can be considerably simplified by use of symmetry properties. Finally one finds, taking in to account the ω terms, as shown in the next subsections, the following result

$$\begin{aligned} \tilde{X} = -2UD_6^R = & -12G(1, 0, -1) + 12G(12, 0, -12) + 36G(14, 0, -14) - 36G(124, 0, -124) \\ & -12G(12, 3, -123) + 12G(1, 23, -123) - 36(4, 1, -14) + 36G(4, 12, -124) \\ & + 36G(12, 4, -124) - 36G(14, 23, 56) + 2 \sum_{i=1}^6 \omega(i) D_6^R. \end{aligned} \quad (110)$$

According to (63) to find X we have to add to this expression twice the term (61), take one half of the sum and subtract D_{60} . This gives our final expression (64) for X in terms of G 's.

7.2 Terms with ω 's

To check the cancellation of all the terms with ω 's we have to write out all of them. We write those in \tilde{X} as Y and we want to see the total amount of them in X

$$Y - 2 \sum_{i=1}^6 \omega(i) D_6^R \quad (111)$$

as given by eq. (50). Then our aim is to see if all terms cancel.

Start with \tilde{X}_1 . It has the following W 's:

$$\begin{aligned} 2W(1, 0, -1) & \rightarrow -2D_2(1)[\omega(0) - \omega(1)] - 2D_2(1)[\omega(0) - \omega(1)], \\ -W(1, 2, -12) & \rightarrow D_2(1)[\omega(2) - \omega(1)] + D_2(12)[\omega(2) - \omega(12)], \\ -W(1, 3, -13) & \rightarrow D_2(1)[\omega(3) - \omega(1)] + D_2(13)[\omega(3) - \omega(13)]. \end{aligned}$$

Adding these new terms to the old ones we have a term with $D_2(1)$:

$$D_2(1)[2 \sum_{i=2}^3 \omega(i) + 4 \sum_{i=4}^6 \omega(i) - 2\omega(23)$$

$$-2\omega(45) - 2\omega(56) - 2\omega(64) - 4\omega(0) + 4\omega(1) + \omega(2) + \omega(3) - \omega(1) - \omega(1)],$$

which together with others and subtracting $2 \sum_{i=1}^6 \omega(i) D_2(1)$ gives

$$\begin{aligned} Y_1 = D_2(1)[\omega(2) + \omega(3) + 2 \sum_{i=4}^6 \omega(i) - 2\omega(23) - 2\omega(45) \\ - 2\omega(56) - 2\omega(64)] + D_2(12)[\omega(2) - \omega(12)] + D_2(13)[\omega(3) - \omega(13)]. \end{aligned} \quad (112)$$

Here we used $\omega(0) = 0$, thanks to the good behaviour due to I.R. cancellations.

Now W 's from \tilde{X}_{12} :

$$\begin{aligned} 2W(12, 0, -12) &\rightarrow -2D_2(12)[\omega(0) - \omega(12)] - 2D_2(12)[\omega(0) - \omega(12)], \\ -W(-12, 1, 2) &\rightarrow D_2(12)[\omega(1) - \omega(12)] + D_2(2)[\omega(1) - \omega(2)], \\ -2W(-123, 3, 12) &\rightarrow 2D_2(123)[\omega(3) - \omega(123)] + 2D_2(12)[\omega(3) - \omega(12)], \\ +W(2, 13, -123) &\rightarrow -D_2(2)[\omega(13) - \omega(2)] - D_2(123)[\omega(13) - \omega(123)], \\ -W(-12, 2, 1) &\rightarrow D_2(12)[\omega(2) - \omega(12)] + D_2(1)[\omega(2) - \omega(1)], \\ W(1, 23, -123) &\rightarrow -D_2(1)[\omega(23) - \omega(1)] - D_2(123)[\omega(23) - \omega(123)]. \end{aligned}$$

Term with $D(12)$ results

$$\begin{aligned} D_2(12)[2\omega(1) + 2\omega(2) + 4 \sum_{i=4}^6 \omega(i) - 2\omega(12) - 2\omega(45) - 2\omega(56) \\ - 2\omega(64) - 4\omega(0) + 4\omega(12) + \omega(1) - \omega(12) + \omega(2) - \omega(12) + 2\omega(3) - 2\omega(12)]. \end{aligned}$$

Summing all and subtracting $2 \sum_{i=1}^6 \omega(i) D_2(12)$ we obtain

$$\begin{aligned} D_2(12)[\omega(1) + \omega(2) + 2 \sum_{i=4}^6 \omega(i) - 2\omega(12) - 2\omega(45) - 2\omega(56) - 2\omega(64)] \\ + D_2(1)[\omega(2) - \omega(23)] + D_2(2)[\omega(1) - \omega(13)] + D_2(123)[2\omega(3) - \omega(13) - \omega(23)]. \end{aligned} \quad (113)$$

Terms from \tilde{X}_{14} The W 's give

$$\begin{aligned} 4W(14, 0, -14) &\rightarrow -4D_2(14)[\omega(0) - \omega(14)] - 4D_2(14)[\omega(0) - \omega(14)], \\ -2W(-14, 1, 4) &\rightarrow +2D_2(14)[\omega(1) - \omega(14)] + 2D_2(4)[\omega(1) - \omega(4)], \\ -W(-124, 2, 14) &\rightarrow +D_2(124)[\omega(2) - \omega(124)] + D_2(14)[\omega(2) - \omega(14)], \\ +W(4, 12, -124) &\rightarrow -D_2(4)[\omega(12) - \omega(4)] - D_2(124)[\omega(12) - \omega(124)], \\ -W(-134, 3, 14) &\rightarrow +D_2(134)[\omega(3) - \omega(134)] + D_2(14)[\omega(3) - \omega(14)], \\ +W(4, 13, -134) &\rightarrow -D_2(4)[\omega(13) - \omega(4)] - D_2(134)[\omega(13) - \omega(134)], \\ -2W(-14, 4, 1) &\rightarrow +2D_2(14)[\omega(4) - \omega(14)] + 2D_2(1)[\omega(4) - \omega(1)], \\ -W(-145, 5, 14) &\rightarrow +D_2(145)[\omega(5) - \omega(145)] + D_2(14)[\omega(5) - \omega(14)], \end{aligned}$$

$$\begin{aligned}
& +W(1, 45, -145) \rightarrow -D_2(1)[\omega(45) - \omega(1)] - D_2(145)[\omega(45) - \omega(145)], \\
& -W(-146, 6, 14) \rightarrow +D_2(146)[\omega(6) - \omega(146)] + D_2(14)[\omega(6) - \omega(14)], \\
& +W(1, 46, -146) \rightarrow -D_2(1)[\omega(46) - \omega(1)] - D_2(146)[\omega(46) - \omega(146)].
\end{aligned}$$

Term with $D_2(14)$ results

$$\begin{aligned}
& D_2(14)[2\omega(2) + 2\omega(3) + 2\omega(5) + 2\omega(6) - 2\omega(23) - 2\omega(56) \\
& \quad - 8\omega(0) + 8\omega(14) + 2\omega(1) - 2\omega(14) + \omega(2) \\
& \quad - \omega(14) + \omega(3) - \omega(14) + 2\omega(4) - 2\omega(14) + \omega(5) - \omega(14) + \omega(6) - \omega(14)].
\end{aligned}$$

Summing all and subtracting $2 \sum_{i=1}^6 \omega(i) D_2(14)$ gives

$$\begin{aligned}
& D_2(14)[\omega(2) + \omega(3) + \omega(5) + \omega(6) \\
& \quad - 2\omega(23) - 2\omega(56)] + D_2(1)[2\omega(4) - \omega(45) - \omega(46)] + D_2(4)[2\omega(1) - \omega(12) - \omega(13)] \\
& \quad + D_2(124)[\omega(2) - \omega(12)] + D_2(134)[\omega(3) - \omega(13)] \\
& \quad + D_2(145)[\omega(5) - \omega(45)] + D_2(146)[\omega(6) - \omega(46)]. \tag{114}
\end{aligned}$$

Terms from \tilde{X}_{124} . From W 's we get

$$\begin{aligned}
& 4W(124, 0, -124) \rightarrow -4D_2(124)[\omega(0) - \omega(124)] - 4D_2(124)[\omega(0) - \omega(124)], \\
& \quad -W(-124, 1, 24) \rightarrow D_2(124)[\omega(1) - \omega(124)] + D_2(24)[\omega(1) - \omega(24)], \\
& \quad -2W(-1234, 3, 124) \rightarrow 2D_2(56)[\omega(3) - \omega(56)] + 2D_2(124)[\omega(3) - \omega(124)], \\
& \quad +W(24, 13, -1234) \rightarrow -D_2(24)[\omega(13) - \omega(24)] - D_2(56)[\omega(13) - \omega(56)], \\
& \quad -W(-124, 2, 14) \rightarrow D_2(124)[\omega(2) - \omega(124)] + D_2(14)[\omega(2) - \omega(14)], \\
& \quad +W(14, 23, -1234) \rightarrow -D_2(14)[\omega(23) - \omega(14)] - D_2(56)[\omega(23) - \omega(56)], \\
& \quad -2W(-124, 4, 12) \rightarrow 2D_2(124)[\omega(4) - \omega(124)] + 2D_2(12)[\omega(4) - \omega(12)], \\
& \quad -W(-1245, 5, 124) \rightarrow D_2(36)[\omega(5) - \omega(36)] + D_2(124)[\omega(5) - \omega(124)], \\
& \quad +W(12, 45, -1245) \rightarrow -D_2(12)[\omega(45) - \omega(12)] - D_2(36)[\omega(45) - \omega(36)], \\
& \quad -W(-1246, 6, 124) \rightarrow D_2(35)[\omega(6) - \omega(35)] + D_2(124)[\omega(6) - \omega(124)], \\
& \quad +W(12, 46, -1246) \rightarrow D_2(12)[\omega(46) - \omega(12)] - D_2(35)[\omega(46) - \omega(35)].
\end{aligned}$$

Term with $D_2(124)$ results

$$\begin{aligned}
& D_2(124)[2\omega(1) + 2\omega(2) + 2\omega(5) + 2\omega(6) - 2\omega(12) - 2\omega(56) \\
& \quad - 8\omega(0) + 8\omega(124) + \omega(1) - \omega(124) + 2\omega(3) - 2\omega(124) + \omega(2) - \\
& \quad \omega(124) + 2\omega(4) - 2\omega(124) + \omega(5) - \omega(124) + \omega(6) - \omega(124)].
\end{aligned}$$

Summing all and subtracting $2 \sum_{i=1}^6 \omega(i) D_2(124)$

$$\begin{aligned}
& D_2(124)[\omega(1) + \omega(2) + \omega(5) + \omega(6) - 2\omega(12) - 2\omega(56)] \\
& \quad + D_2(12)[2\omega(4) - \omega(45) - \omega(46)] + D_2(14)[\omega(2) - \omega(23)] \\
& \quad + D_2(24)[\omega(1) - \omega(13)] + D_2(35)[\omega(6) - \omega(46)] \\
& \quad + D_2(36)[\omega(5) - \omega(45)] + D_2(56)[2\omega(3) - \omega(13) - \omega(23)]. \tag{115}
\end{aligned}$$

From the last term \tilde{X}_{123} we trivially find

$$Y_{123} = D_2(123)[2 \sum_{i=1}^6 \omega(i) - 2\omega(12) - 2\omega(23) - 2\omega(31) - 2\omega(45) - 2\omega(56) - 2\omega(46)]. \quad (116)$$

To see that all these ω terms cancel we have to take into account the symmetries and study the sum

$$6Y_1 - 6Y_{12} - 9Y_{14} + 9Y_{124} + Y_{123}. \quad (117)$$

Summing all the Y 's and making the necessary permutations we get a sum of five different structures

$$a_1 D_2(1) + a_{12} D_2(12) + a_{14} D_2(14) + a_{124} D_2(124) + a_{123} D_2(123).$$

It is then straightforward to see that all the coefficients a_1 , a_{12} , a_{14} , a_{124} and a_{123} are equal to zero. So indeed all ω terms in X cancel.

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